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Neighborhood Effects in Wind Farm Performance: An Econometric Approach

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Neighborhood Effects in Wind Farm Performance: An Econometric Approach*

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Abstract

The optimization of turbine density in wind farms entails a trade-off between the usage of scarce, expensive land and power losses through turbine wake effects. A quantification and prediction of the wake effect, however, is challenging because of the complex aerodynamic nature of the interdependencies of turbines. In this paper, we propose a parsimonious data driven econometric wake model that can be used to predict production losses of existing and potential wind parks. Motivated by simple engineering wake models, the predicting variables are wind speed, turbine alignment angle, and distance. By utilizing data from two wind parks in Germany, a significantly better prediction of wake effect losses is attained compared to the standard Jensen model. A scenario analysis reveals that a distance between turbines can be reduced up to three times the rotor size without entailing substantial production losses. In contrast, a suboptimal configuration of turbines with respect to the main wind direction can result in production losses that are five times higher.

Keywords: Wind energy; wake modeling; wind farm design.

JEL codes: Q42; Q47.

Introduction

Turbine density is a crucial parameter in designing the layout of wind farms. Optimizing turbine density must consider the trade-off between two productivity indicators: the generated energy per turbine and the generated energy per area. In most industrialized countries, land that can be used for wind energy production is scarce and costly. This scarcity calls for a high turbine density per area. On the other hand, it is well-known that as turbine density increases, the productivity per turbine decreases due to wake effects of neighboring turbines, i.e., each turbine absorbs part of the incoming wind energy, reduces wind speed, and creates turbulences in the downstream direction. Quantification of this wake effect, however, is challenging because of the complex aerodynamic nature of the interdependencies of turbines within a wind farm.[1] Many attempts have been made to estimate wind power losses by means of wake effect models. Crespo et al. (1999) and Barthelmie and Pryor (2013) provide an overview about the various modeling approaches. Kiranoudis and Maroulis (1997) distinguish between explicit (kinematic) wake models that provide closed-form expressions for the velocity deficit and implicit models (field models) that are based on approximations of Navier-Stokes or vorticity equations. Examples of the first model type are the Jensen model (Jensen 1983), the Frandsen model (Frandsen 1992), and the Larsen model (Larsen 1988). The second model class is comprised of the Eddy-viscosity by Ainslie (1985), the Fuga

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[1] In this article, we focus on neighborhood effects within a wind park. Another strand of literature considers interdependencies among wind farms on a larger regional scale (e.g., Miller et al. 2015, Adams and Keith 2013).

model by Ott et al. (2011), and the large eddy simulation models (Witha et al. 2014). These models differ largely in their complexity and computational burden.

Once a wake model has been chosen and specified, it can be used to optimize the layout of wind farms, which aims to optimally position turbines such that expected power production is maximized (Asta 2013). Several optimization procedures have been suggested to solve the layout problem, including mathematical programming (e.g., Turner et al. 2014), Monte Carlo Simulation (Marmidis et al. 2008; Brusca et al. 2014), or multi-objective programming (Kuziak and Song 2014). Most of the aforementioned approaches for optimizing wind farm layout resort to simple wake models, particularly the Jensen model (c.f. Bastankhah and Porté-Agel 2014). The reason is that the Jensen model expresses the velocity deficit vd due to the wake of a (single) turbine as a simple function of the downstream rotor radius r_d, the downwind distance d, the axial induction factor a, and a measure of velocity of wake expansion η (Samorani 2013), given by:

$$vd = \frac{2a}{1 + \eta \left(\frac{d}{r_d}\right)^2} \tag{1}$$

The wind deficit vd can then be directly incorporated into the objective function of optimization procedures. Eq. (1) constitutes a single wake model, but it is rather easy to extend it to a multi-wake model. The single wakes of turbines j that affect turbine i, $W(i)$, can be aggregated, for example, through a Euclidean norm to obtain the total velocity deficit of a turbine in position i as[2]:

$$VD_i = \sqrt[2]{\sum_{j \in W(i)} vd_j^2} \tag{2}$$

The second reason for the widespread use of simple kinematic models is the fact that they are considered as fair approximations to statistically measured power losses despite their simplicity (Pena et al. 2015). This view, however, has been challenged. Bastankhan and Porté-Agel (2014) criticize the top hat distribution of the velocity deficit implied by the Jensen model and replace it by a Gaussian distribution to attain a better fit to experimental data. Gaumond et al. (2012) report that for two off-shore wind parks, the Jensen model underestimates production in the first few turbine rows of the wind parks, yet it overestimates production in the last few turbine rows. Wu et al. (2012) propose a generalization of the multi-wake model (2) that considers a partial overlap of the wake area and the rotor swept area.

In this paper, we propose an alternative method to estimate power losses from wake effects for a wind park. Instead of deriving a functional form for the velocity deficit from aerodynamic relations, we pursue an empirical, data-driven approach, which links the basic determinants of velocity losses, such as entering wind speed, turbine alignment angle, and distance between turbines, in a flexible way and applies econometric techniques to estimate the parameters of the model. The advantage of this proposed approach is that it relies on a few assumptions only and it can be adapted to any wind park design. In this paper, we demonstrate the specification and estimation of the regression model for two wind parks in Germany. We find that our model is a good fit with observed wind deficits and outperforms the Jensen model. Depending on the location of the turbine, average production losses per turbine due to wind wakes are between 0.9 and 14.6% of the turbine's potential power production in one year. We also find that the positioning of the turbines within the wind park is more important than the turbine density.

[2] Alternative wake adding methods are described in Göcmen et al. (2016), for example.

This is not the first time that turbine interaction effects in wind farms have been estimated with econometric methods. Alexiadis et al. (1999) were among the first to consider neighboring effects when predicting wind speed and power output of a wind farm. Xie et al (2011) propose a spatio-temporal wind forecast model that accounts for spatial wind power correlations. Recently, Croonenbroeck and Ambach (2015) estimate power production of a wind farm using a spatial lag model with panel data. Wake effects are implicitly captured by an interaction parameter. The contribution of our paper to the existing literature is that we focus on power losses within an existing wind park. Moreover, we aim to explain observed power losses by means of fundamental variables. In contrast to other econometric models, we explicitly take into account the relative position of individual turbines in a wind farm. Hence, our approach can be considered as a hybrid model that combines elements of engineering wake models and empirical models.

Methods

We proceed in several steps to estimate the power loss of a wind park due to wake effects. First, we estimate the wind velocity deficit of a single turbine, which is affected by one or more turbines in the wind park. To this end, we approximate aerodynamic engineering models with a simple econometric model, which allows for the inclusion of multiple turbine wake effects, but is parsimonious in its specification and is applicable to any wind park design. The resulting velocity deficit is then translated to a power loss via a given power curve. We do not directly explain power losses since they might be caused by factors other than wake effects. Finally, the power losses of all individual turbines in a wind park are aggregated. In what follows, we describe this procedure in detail.

We model the velocity deficit of a turbine i at time t, VD_{it}, as a function of three variables that are suggested by engineering wake models. These three variables are: the entering (undisturbed) wind speed, w_t^{∞}; the distance between a disturbing turbine j and a disturbed turbine i, d_{ij}; and the turbine alignment angle between a disturbing turbine j and a disturbed turbine i, $\alpha_{ij,t}$. The turbine alignment angle describes the deviation from the situation that the straight line formed by the turbine row coincides with the wind direction (McKay et al. 2012). Hence, an angle of 0° means that the disturbing turbine stands directly in front and causes a maximum wake effect to the disturbed turbine. The higher the angle $\alpha_{ij,t}$, the lower the disturbance of turbine i by turbine j, keeping the distance d_{ij} fixed. Due to the dependence on the wind direction, turbine alignment angles depend on time and the position of the turbines. The undisturbed wind speed is the same for all turbines and only depends on time t, whereas the distance between turbines i and j is determined by the position of the turbines.

To measure the effect of relevant turbines in every moment and to allow for the observations to be comparable, we do not consider fixed neighbors. Instead, we sort neighboring turbines according to their impact on the disturbed turbine. Otherwise, with fixed neighbors we would only find the average effect of each turbine, which is specific to a particular wind park design and wind conditions and therefore cannot be generalized. To sort relevant neighbors, we first delimit the radius of potential influence of the disturbing turbines to 1 km. This distance represents approximately 10 to 12 times the rotor diameters of typical turbines and most kinematic models predict negligible losses for larger distances. Then, we sort the remaining turbines by their turbine alignment angle, i.e., the neighbor with the lowest angle is assumed to be the most disturbing one. Here, we do not distinguish if the turbine stands on the left or right side compared to the wind direction.[3] As an alternative to this sorting procedure, one could use a more sophisticated combination of the distance and angle to find the most disturbing turbine(s). However, for this purpose we would already have to use a specific wake model, which we want to derive at a later stage.

Fig. 1 depicts the case of one turbine disturbed by three neighbors within a 1 km radius given a certain wind direction. To simplify notation, from now on we will omit the index of the disturbed turbine i and time t. Fig. 1 illustrates the angles $\alpha_1 - \alpha_3$ and distances $d_1 - d_3$ from the disturbing turbines T_1–T_3 to the disturbed turbine T_0. In this moment t, the wind comes from the left (black arrows) and we consider the loss of turbine T_0. In this example, turbine T_1 is assumed to be the most disturbing neighbor since the angle α_1 is the smallest. Turbine T_2 depicts the second most disturbing turbine, whereas the third neighboring turbine T_3 is too laterally situated to affect turbine T_0 given the current wind direction, i.e., the turbine alignment angle α_3 is too large compared to the other neighbors, no matter the size of distance d_3.

This sorting depends on the current wind direction, i.e., it is different for a different wind direction at another time t. This implies that the distances to the (relative) neighbors also become time dependent. Moreover, we calculate the wake effect loss separately for each turbine using the same sorting procedure. If we consider the loss of turbine T_3 in Fig. 1, for example, T_2, T_1, and T_0 are the first, second, and third most disturbing neighbors for this wind direction, respectively.

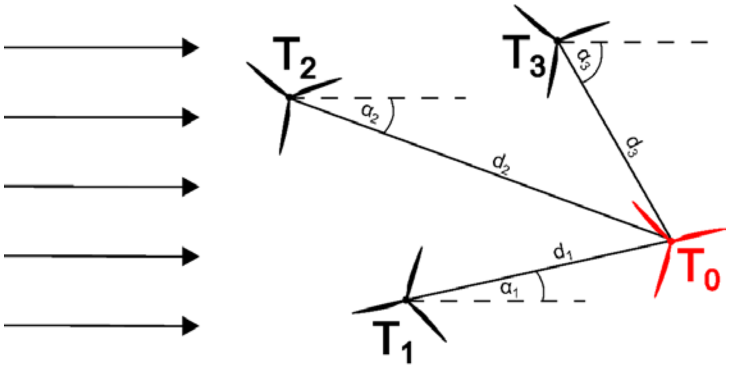

Figure 1: Definition of distance and turbine alignment angle in the regression model

We use a fully interacted linear regression model to relate the velocity deficit to the explanatory variables. In our models, we confine the analysis to one or two disturbing neighboring turbines. Samorani (2013) shows that the inclusion of more than the closest disturbing turbine does not have a significant effect on the velocity deficit, i.e., the marginal effect of more than one turbine is low. In case of a single disturbing turbine (i.e., a single wake model), the specification of the model for the losses of turbine T_0 is as follows:

$$\begin{aligned} VD_0 &= f(\alpha_1, d_1, w^\infty) \\ &= \beta_1\alpha_1 + \beta_2 d_1 + \beta_3\alpha_1 d_1 + \beta_4\,\alpha_1 w^\infty + \beta_5\,d_1 w^\infty + \beta_6\alpha_1 d_1 w^\infty + \beta_7 w^\infty + \epsilon, \end{aligned} \tag{3}$$

where $\beta_1 \ldots \beta_7$ are parameters to be estimated for the most disturbing neighboring turbine and ϵ denotes the error term. In case of two disturbing neighbors, a two-wake model can be constructed by adding a similar expression as in Eq. (3) for the second turbine:

[3] Formally, this means that we consider the absolute value of the angles.

$$
\begin{aligned}
VD_0 &= f(\alpha_1, d_1, \alpha_2, d_2, w^{\infty}) \\
&= \beta_1\alpha_1 + \beta_2 d_1 + \beta_3\alpha_1 d_1 + \beta_4\,\alpha_1 w^{\infty} + \beta_5\,d_1 w^{\infty} + \beta_6\alpha_1 d_1 w^{\infty} + \beta_7 w^{\infty} \\
&\quad + \beta_8\alpha_2 + \beta_9 d_2 + \beta_{10}\alpha_2 d_2 + \beta_{11}\,\alpha_2 w^{\infty} + \beta_{12}\,d_2 w^{\infty} + \beta_{13}\alpha_2 d_2 w^{\infty} + \epsilon.
\end{aligned}
\tag{4}
$$

In the two-wake-model, the linear wind variable enters the model only once to avoid multicollinearity. The use of a simple linear regression model may appear restrictive at first glance in contrast with engineering wake models; however, the inclusion of interaction terms allows for the impact of the three explanatory variables to be nonlinear. Using interaction terms is also advisable because the three variables unfold their impact jointly. For example, a low distance and low angle are only relevant if there is wind. In the same manner, the wind speed passing through two different disturbing turbines has a different interacted effect on the wind deficit of the disturbed turbine. Once the model parameters have been estimated, standard statistical procedures can be applied to test the significance of the parameter estimates and the model's goodness of fit.

Even though we are interested in economic losses caused by turbine interaction in a wind park, we first estimate wind velocity deficits rather than power losses. The reason is that power losses may be caused by reasons other than wakes of other turbines. Pieralli et al. (2015) show that empirically observed productivity losses are often rooted in various kinds of turbine errors, such as icing or maintenance. Since it is difficult to disentangle these effects without detailed information on the error status of turbines, we prefer to derive power losses by inserting observed (or estimated) wind velocity deficits into the turbine's power curve.

The aggregation of losses to the level of the wind park is carried out by summing the predicted losses for all turbines occurring in every moment in a wind park. The results of the regression model can be used to predict losses for hypothetical wind park designs. Based on historical wind speeds and absolute wind directions observed every 10 minutes, we calculate distances and turbine alignment angles for each turbine, considering its position within the wind park. For every turbine, we then decide on the number of disturbing neighbors. If there is no disturbing neighbor, we set the wind deficit and power loss to zero. For one or two disturbing turbines, plugging distances, angles, and wind speeds into the corresponding estimated wake regression model (a single-wake or two-wake model) yields the wind velocity deficit for the disturbed turbine in a particular moment. Undisturbed wind speeds and turbine-specific disturbed wind speeds are then plugged into the power curve to obtain wake effect power losses. This calculation is repeated for all turbines in the wind park in every moment and then summed for the whole wind park. These losses can be further aggregated by turbine to study the losses of different turbines within the park.

Empirical Application

We apply the estimation procedure described above to two onshore wind farms located in Bavaria and Saxony-Anhalt in Germany. They are comprised of seven turbines of type Enercon E-82 with a capacity of 2.3 MW, a hub height of 138 m, and a diameter of 82 m (hereafter, wind park A) and of twelve turbines of type Fuhrländer FL2500 with a capacity of 2.5 MW, a hub height of 100 m, and a diameter of 100 m (hereafter, wind park B). The positioning of turbines in the two parks are depicted in Fig. 2. Wind farm A is located along the southwest-northeast direction and turbines have an average distance of 713 meters (equivalent to 8.7 rotor diameters). Wind farm B is located in a rectangle with an average distance of 657 meters (equivalent to 6.6 rotor diameters). Due to these differences, it can be expected that both wind farms have different wake effects even under similar topographic conditions. In particular, we conjecture that wind farm A is more sensitive to changes in the wind direction than wind farm B.

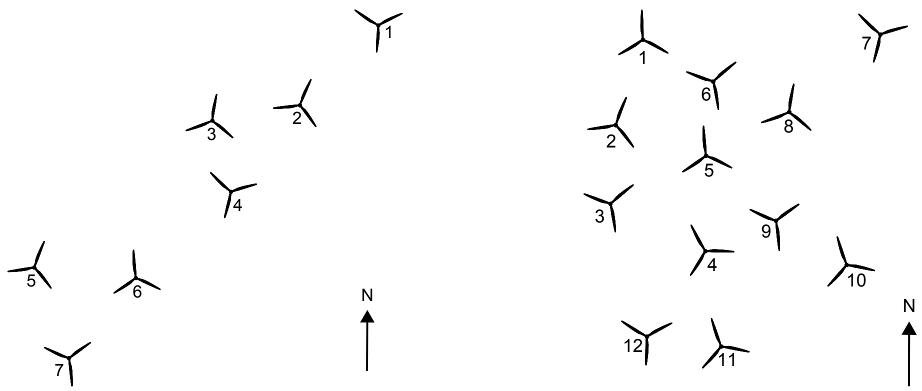

Figure 2: Turbine configuration of wind park A (left) and B (right)

Data

Data for both wind parks are comprised of the exact coordinates for each turbine and their Supervisory Control and Data Acquisition (SCADA) data.[4] SCADA data have the advantage of providing turbine-specific information on a low temporal scale. This is particularly useful for wake effect analyses (Barthelmie and Pryor 2013). Moreover, SCADA data are true observations, so the wake effect can be studied under realistic conditions. SCADA data contain average wind speeds over 10 minutes for each turbine for the time period 2014.01.01–2014.03.31 (wind park A) and 2011.11.14–2013.12.31 (wind park B). For wind park A, we also have data on the nacelle position of each turbine, which we average for each moment to obtain a proxy for wind direction.[5] For wind park B, no direction data are available, so we match the observed SCADA data with reanalysis wind direction data from Modern Era Retrospective-Analysis for Research and Applications (MERRA, cf. Rienecker et al. 2011). Turbine alignment angles and distances are then calculated dependent on the wind speed for every moment and every turbine. For the data on the undisturbed wind speed, we take the maximum mast wind speed among all turbines for every moment since there is no undisturbed meteorological mast available. This is a reasonable approximation because at least one turbine should always not be disturbed by a wake effect and is therefore supposed to have the highest mast wind speed in the wind park.

To obtain a rather homogenous dataset and to measure the pure wake effect, we perform several data cleaning steps. First, we exclude all observations where the undisturbed mast wind speed is below 4 m/s or above 14 m/s, i.e., the rather constant parts in the power curves. For these observations, the wake effect is close to zero since wind deficits do not lead to production differences: The observed wind speeds lie either below the cut-in speed or above the rated wind speed where maximum production is reached. Second, only those observations are considered where wind speed data for all turbines are available. Third, since we want to study wind speed deficits caused by the wake effect, we exclude all observations where the turbine alignment angle with the first or second most disturbing

[4] The authors cordially thank 4initia GmbH for providing the data.
[5] Gaumond et al. (2013) point out that the use of the nacelle position may cause wind direction uncertainty due to a yaw misalignment of the turbine.

neighbor is higher than ±30°.[6] This is a rather large range (we do not consider the direction of the angle, so it is 30° to the left or 30° to the right) compared to other studies. Croonenbroeck and Ambach (2015), for example, restrict their analysis to observations within ±10°.

We end up with a total of 11,670 (wind park A) and 418,746 (wind park B) turbine-specific observations. Table 1 presents summary statistics of the data used in the regression. The average undisturbed wind speeds are 8.5 (wind park A) and 8.8 m/s (wind park B). As a result of our data confinement, the reported averages are not representative for the wind parks' average wind speeds. The average turbine alignment angle lies at 9.2° (wind park A) and 8.2° (wind park B) for the first neighbor and at 17.7° (wind park A) and 17.8° (wind park B) for the second neighbor. By construction, the angle of the second turbine is always larger than that of the first turbine. Moreover, the maximum angle is 30° and maximum distance is 1 km according to our selection procedure. Differences in the distance between the first and second turbine are negligible: 0.713 km compared to 0.715 km for wind park A and 0.657 km compared to 0.647 km for wind park B. We see, however, that the average distance in wind park B is lower than that in wind park A, indicating a higher turbine density in wind park B.

Table 1: Summary statistics of the wind farm data used in the regression

	Mean	Std. Dev.	Min	Max
Wind Park A (11,670 observations)				
Velocity Deficit (m/s)	1.153	0.939	0.000	9.700
Angle 1st neighbor (º)	9.291	7.366	0.087	29.608
Distance 1st neighbor (km)	0.713	0.219	0.326	0.963
Angle 2nd neighbor (º)	17.694	8.190	0.392	29.813
Distance 2nd neighbor (km)	0.715	0.207	0.326	0.963
Undisturbed Wind speed (m/s)	8.518	2.151	4.400	14.000
Wind Park B (418,746 observations)				
Velocity Deficit(m/s)	1.659	1.238	0.000	9.889
Angle 1st neighbor (º)	8.226	5.968	0.000	29.926
Distance 1st neighbor (km)	0.657	0.187	0.320	1.000
Angle 2nd neighbor (º)	17.809	7.089	0.300	30.000
Distance 2nd neighbor (km)	0.647	0.209	0.320	1.000
Undisturbed Wind speed (m/s)	8.824	2.144	4.033	13.999

In both wind parks, for most of the time the dominating absolute wind direction is south-west (between 210º and 270º). As a representative display of the wind, Fig. 3 depicts the wind rose distribution for wind park B in the calendar year 2011. Because of its representativeness of the wind conditions encountered in wind parks A and B, later we will apply this historical wind distribution to predict losses for hypothetical wind park designs.

[6] One could use a larger dataset for the single-wake model by not excluding observations where the turbine alignment angle of the second neighbor is larger than 30°; however, this would lead to different datasets for the models and would not allow comparable results.

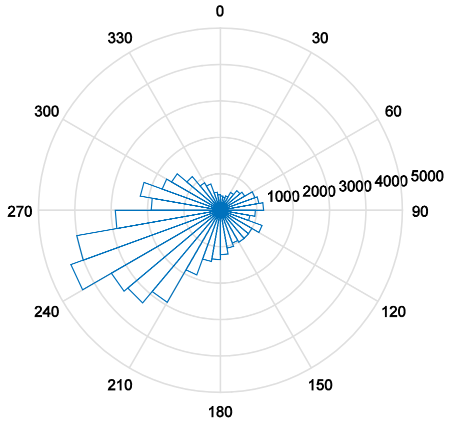

Figure 3: Histogram of wind direction in 2011

Results

Regression models (3) and (4) have been estimated separately for the two wind parks with Ordinary Least Squares (OLS) using STATA. For both wind farms, we specify a single-wake model and a two-wake model. These models are conservative in estimating the wind deficit between the undisturbed wind speed and the disturbed wind speed in the interior of wind park A or B since greater turbine alignment angles than we consider (30º) and higher numbers of neighbors might additionally cause wake effect wind deficits. The estimation results are presented in Table 2. Estimated coefficients of determination, R^2, of about 0.7 indicate that the overall fit of the models is fairly good for both wind farms.

Note that the performance of the two-wake model is only slightly better than the single-wake model, i.e., the highest explanatory power comes from the most disturbing turbine. For the single-wake models, most of the coefficients are significant at the 1% level. This is also true for the two-wake model in the case of wind farm B. In contrast, 6 of the 13 coefficients are not significant for wind farm A. There are also differences between the four models regarding the size and sign of the estimated parameters. While the main effect of the distance of the first neighbor is negative for all model variants, the main effect of the angle of the first neighbor is negative only for wind farm B. Contrary to expectations, the main effect of this angle is positive for wind farm A. However, in view of the three-way interaction effects, it is difficult to assess the impact of the explanatory variables by inspection of the coefficients of the separate main effects.

To attain a better understanding of the influence of the wind speed, the turbine alignment angle, and the distance of disturbing neighboring turbines on the velocity deficit, we calculate the marginal effects of these variables while holding the other variables at their sample means (see Table 3). In line with what can be conjectured from engineering models, we find that the average marginal effect of the entering undisturbed wind speed is positive, whereas the turbine alignment angle and the distance of the most disturbing turbine have a negative effect on the wind deficit. In other words, a higher turbine alignment angle results in a lower predicted wind deficit, whereas a greater distance from the disturbing neighbor lowers the predicted wind deficit. A somewhat surprising result is the positive marginal effect of the distance of the second turbine. This might be explained by the fact that we select the most disturbing turbines by considering their angle to the turbines they are disturbing, but we disregard the distance between them.

Table 2: Estimation results: regression coefficients

	Wind Park A				Wind Park B			
	Single-Wake		Two-Wake		Single-Wake		Two-Wake	
	Coeff.		Coeff.		Coeff.		Coeff.	
Angle1	0.019	***	0.001		-0.031	***	-0.036	***
Distance1	-0.823	***	-0.794	***	-0.610	***	-0.557	***
Angle1 * Distance1	0.015		0.019		0.069	***	0.063	***
Wind	0.225	***	0.245	***	0.222	***	0.246	***
Angle1 * Wind	-0.008	***	-0.006	***	-0.001		0.002	***
Distance1 * Wind	0.036	***	0.038	*	0.052	***	0.051	***
Angle1 * Distance1 * Wind	-0.0003		-0.001		-0.007	***	-0.007	***
Angle2			0.019	**			0.012	***
Distance2			-0.510	***			-0.493	***
Angle2 * Distance2			0.010				0.013	***
Angle2 * Wind			-0.005	***			-0.005	***
Distance2 * Wind			0.034				0.044	***
Angle2 * Distance2 * Wind			0.001				-0.001	
R^2	0.723		0.727		0.702		0.704	

Notes: *, **, and *** denote statistical significance at the 10, 5, and 1 percent level, respectively.

Table 3: Estimation results: average marginal effects (changes due to 1% change in variables)

	Wind Park A				Wind Park B			
	Single-Wake		Two-Wake		Single-Wake		Two-Wake	
	Coeff.		Coeff.		Coeff.		Coeff.	
Angle1	-0.403	***	-0.346	***	-0.200	***	-0.148	***
Distance1	-0.278	***	-0.246	***	-0.084	***	-0.067	***
Angle2			-0.096	***			-0.221	***
Distance2			0.108	***			0.061	***
Wind	1.457	***	1.483	***	1.941	***	1.939	***

Notes: *, **, and *** denote statistical significance at the 10, 5, and 1 percent level, respectively.

The results of the empirical wake models are graphically displayed in Figures 4 and 5. Fig. 4 depicts the wake effect for both wind parks when two disturbing turbines are positioned next to each other (i.e., at an angle of 90 degrees) at a distance of 500 meters, whereas Fig. 5 presents the simulated wake effects for two disturbing turbines allocated at a distance of 200 meters in a line (i.e., at an angle of zero degrees). Both figures display the estimated wind deficit for a third, disturbed turbine at an entering wind speed of 7 m/s coming from the left. For positions where the turbine alignment angle to one disturbing turbine is lower than 30°, the results from the single-wake model are applied. For overlapping positions where angles to the first and second turbine are below 30°, the two-wake model is used with a sorting of neighbors by exact angles.

The maximal wind deficit in Fig. 4 amounts to approximately 1.5 m/s, which is a reduction of about 20 percent. The maximal wind deficit occurs directly behind the disturbing turbines. The wake effect fades out quickly with increasing distance and turbine alignment angle (i.e., a vertical movement in the graph). Comparing the two wind parks in Fig. 4 reveals that the general behavior of the wake effects is similar with slight differences. For example, in wind park B the wake effect is more intense directly behind the turbines for larger distances (until approximately 300 meters) and persists longer with increasing distance compared to wind park A. These differences can be explained by factors for which we do not control for in the regression approach, in particular, technical parameters such as rotor size (82 m versus 100 m) and hub height (138 m versus 100 m) as well as differences in the wind park terrain. These differences manifest themselves in the specific regression coefficients that we estimate for each wind park separately.

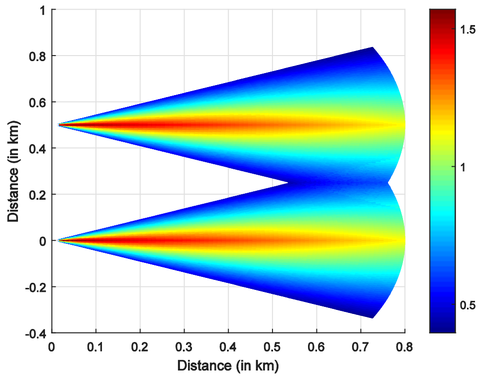

Figure 4: Simulation of wake effect with two lateral turbines for an undisturbed wind speed of 7 m/s coming from the left, wind park A (left) and wind park B (right)

Fig. 5 shows that the wake effect is aggravated by a second disturbing turbine, but that the impacts of the two disturbing turbines are not additive. As before, the wind deficit declines with increasing distance and turbine alignment angle. Similar to Fig. 4, the wake effects endure longer in wind park B when increasing the downstream distance. Moreover, Fig. 5 reveals a flaw in our model: The estimated wind deficit shows a discontinuity when switching from the single to the two-wake model, while in reality the transition is expected to be smooth. This could be overcome by using a smoothening combination of the models in the transition areas.

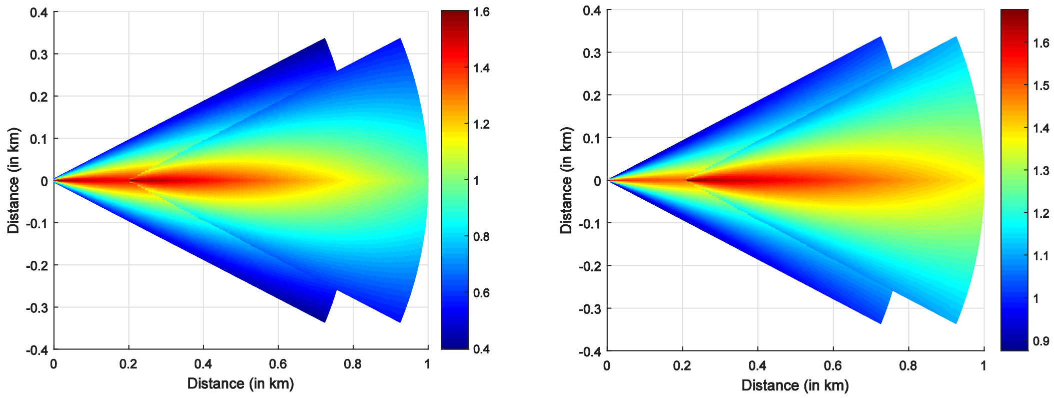

Figure 5: Simulation of wake effect with two consecutive turbines for an undisturbed wind speed of 7 m/s coming from the left, wind park A (left) and wind park B (right)

To assess the performance of the proposed econometric approach, we compare the outcome of our models with the results from a benchmark model, i.e., the model from Jensen in Eq. (1). This requires the specification of the thrust coefficient, the hub height, and the surface roughness. The thrust coefficient for the Enercon E-82 turbine is around 0.8 for our most frequent wind speed. Referring to the guidelines of the World Meteorological

Organization (2010), we chose a surface roughness length of 0.03, which corresponds to open flat terrain, grass, and few isolated obstacles.

For a direct comparison of our econometric model with the Jensen model, we calculate the root mean squared error (RMSE), i.e., the average deviation of the predictions of both models from the true values. The RMSE of our model amounts to 0.78 m/s for wind park A and to 1.13 m/s for wind park B. The errors for the Jensen model are 1.04 m/s for wind park A and 1.30 m/s for wind park B, which are 34% and 16% higher, respectively, than the RMSEs from our model. From these differences, it becomes clear that the econometric approach with a rather simple function (i.e., linear with interactions) outperforms the frequently used Jensen model, at least in this simple version.

The wind deficits, however, are not the best measure to describe the economic losses caused by wake effects since the same wind loss has a different effect on the power production for different intervals of the power curve, depending on its slope. Hence, we also calculate the resulting power losses by plugging the disturbed and undisturbed wind speeds into the power curve. The RMSE for the power losses from the econometric model are 217.39 kW for wind park A and 394.12 kW for wind park B, whereas those for the Jensen model are 300.92 kW (wind park A) and 470.02 kW (wind park B). Thus, the errors for the Jensen model are 38% and 19% higher.

To analyze the above differences in detail, Fig. 6 depicts the observed and predicted power losses against wind speed for an (average) single turbine in wind park B. For isolating the change in losses due to wind speed, we fix the other model variables at their mean values. For every observation, i.e., for every moment and every turbine, we verify whether no, one or two disturbing turbines are relevant, apply the appropriate regression model to calculate the predicted losses, and average the results for a given value of the undisturbed wind speed. Fig. 6 shows that the regression model fits the observed losses fairly well, particularly for wind speeds below 13 m/s. Below 11 m/s, the regression model slightly overestimates the actual losses, whereas it slightly underestimates them between 11 m/s and 13 m/s. Above 13 m/s, the regression model underestimates the observed losses with a gap that increases with higher undisturbed wind speed. However, the regression model outperforms the Jensen model at each level of wind speed. In fact, the Jensen model underestimates the actual power losses significantly, at least for the chosen specification of the parameter in the Jensen model. Fig. 6 also illustrates that the regression model is able to reproduce the decrease in power losses above a wind speed of 11 m/s, which comes from the fact that the power curve flattens when approaching the rated wind speed.

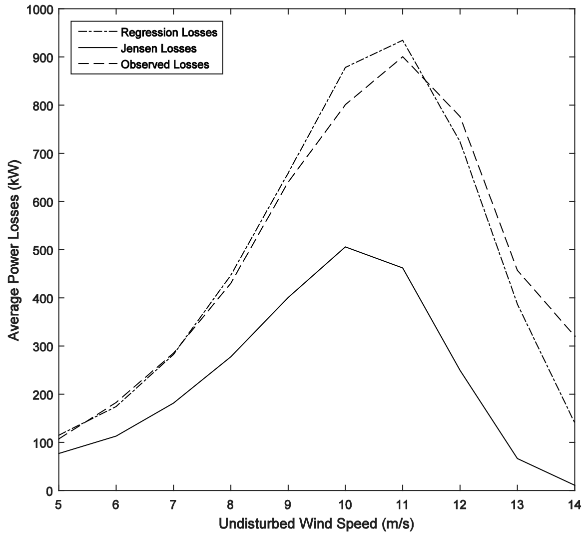

Figure 6: Wake losses as a function of wind speed, wind park B

Scenario Analysis

In this section, we use the regression model to simulate power losses for hypothetical wind park designs with three turbines, given the wind speed and direction from 2011 (see Fig. 3). In the first two scenarios, we investigate the predicted change in wind deficits if the turbine density of a wind park is increased, i.e., the distance between the turbines is reduced (see Fig. 7). Scenario 1 depicts three wind turbines that are 500 m apart and located on the vertices of an equilateral triangle. Scenario 2 reduces the distance between the turbines from 500 m to 250 m. The other two scenarios analyze the impact of the alignment of wind turbines along a specific wind direction. We consider two extreme cases: Scenario 3 presents a row of three turbines aligned in a southwest-northeast direction (245°), which is the main wind direction according to the 2011 data (see Fig. 3). Scenario 4 aligns the three turbines orthogonal to the main wind direction. To evaluate the losses corresponding to these four hypothetical wind park designs, we use the estimated regression coefficients of wind park A and simulate wind deficits for the historical wind data from 2011. We choose wind park A because its coefficient of determination is higher than that in wind park B and hence the model has a better fit to the empirical data.

The results of the hypothetical scenario analyses are presented in Fig. 8 for each turbine separately. When passing from scenario 1 to scenario 2, i.e., when increasing the density of turbines, we see an increase in turbine-specific losses. As one would expect given our regression model and the equidistant position of the turbines in these two scenarios, the increase in losses is similar for all turbines.

What appears more relevant from our analysis is the appropriate choice of the alignment direction. When changing the alignment of the turbines from being in line with the main wind direction to orthogonal to the main wind direction (i.e., passing from scenario 3 to scenario 4), the losses are dramatically reduced. However, there are differences between the turbines: Turbine 1 has an excellent position regarding the main wind direction in both scenarios, so that the losses are almost equal, yet the losses for the other two turbines strongly decrease under optimal alignment. It is also worth noting that the loss of the middle turbine is always the greatest. This means that in scenario 3, it is worse for a turbine to be disturbed from two neighbors at two opposite sides (turbine 2) than it is for a turbine to stand behind two turbines towards the main wind direction (turbine 3).

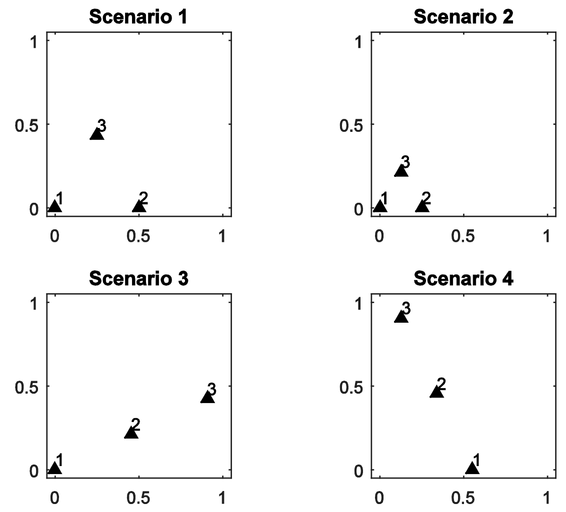

Figure 7: Layout of the hypothetical wind park designs

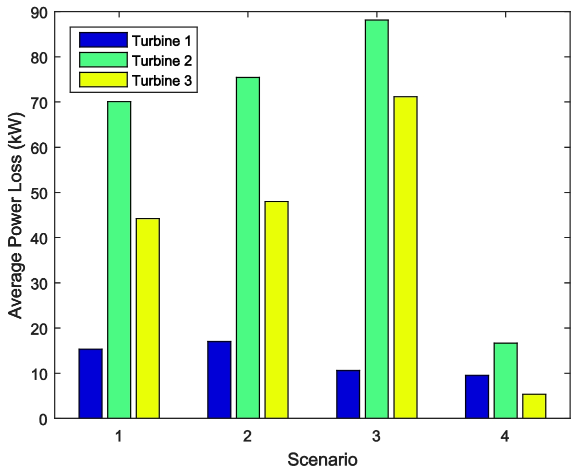

Figure 8: Scenario analysis

Table 4 depicts the losses estimated for the four wind park designs. The first part shows the average 10 min losses per observation as in Fig. 8. The average losses per observation over all turbines are 43 kW (scenario 1), 47 kW (scenario 2), 57 kW (scenario 3), and 11 kW (scenario 4). This stresses again that in our scenarios, the alignment with respect to the wind direction distribution is more important than the density of the turbines.

The middle section of Table 4 shows the total annual loss for the same scenarios, which means that the values per 10 min observation in kW are aggregated for the entire year and converted into MWh. These values are then compared to the maximum attainable yearly production of 5,219 MWh per turbine or 15,658 MWh for all three, i.e., the hypothetical case that all three turbines are not disturbed through wake effects. Increasing turbine density from scenario 1 to scenario 2 leads to changes in losses of less than 50 MWh per turbine, i.e., less than one percentage point. The worst alignment (scenario 3) leads to a loss of almost 10% for the whole wind park. This can be reduced to 1.7% with optimal alignment (scenario 4).

The last part of Table 4 then translates these numbers into monetary amounts by assuming an average price of 32.78 €/MWh, which corresponds to the average European Energy Exchange (EEX) spot market electricity price in 2014. In the worst case, a bad position of a turbine can lead to an extra loss of almost 20,000€ per year for one turbine (turbine 2 from scenario 3 to 4) and a poor wind park design can result in an extra loss of almost 40,000€ per year for the entire wind park (from scenario 3 to 4).

In summary, our analysis shows that the most important decision for wind park design is turbine alignment. The choice of turbine alignment seems to be a more crucial decision than their density, at least for reasonable intra-turbine distances (approximately 2.5 to 3 times the turbine rotor diameter).

Table 4: Estimated losses for hypothetical wind parks in different scenarios

Average 10 min Loss (in kW)	Turbine			
Scenario	1	2	3	All
1	15.4	70.1	44.2	43.2
2	17.0	75.4	48.0	46.8
3	10.6	88.1	71.2	56.6
4	9.5	16.7	5.4	10.5

Total Annual Loss (in MWh)	Turbine			Wind Park
Scenario	1	2	3	
1	132.7	606.0	382.1	1120.9
	(2.5%)	*(11.6%)*	*(7.3%)*	*(7.2%)*
2	147.1	652.1	415.3	1214.5
	(2.8%)	*(12.5%)*	*(8.0%)*	*(7.8%)*
3	91.8	762.0	615.4	1469.3
	(1.8%)	*(14.6%)*	*(11.8%)*	*(9.4%)*
4	82.4	144.0	46.5	273.0
	(1.6%)	*(2.8%)*	*(0.9%)*	*(1.7%)*
Attainable total prod. *(100%)*	5,219.3	5,219.3	5,219.3	15,657.9

Total Yearly Loss (in EUR, 32.78€ per MWh)	Turbine			Wind Park
Scenario	1	2	3	
1	4,351.5	19,864.8	12,526.3	36,742.6
2	4,821.4	21,376.9	13,612.9	39,811.2
3	3,010.6	24,978.7	20,174.1	48,163.3
4	2,702.2	4,721.8	1,524.2	8,948.2

Discussion and Conclusions

In this paper, we propose a parsimonious econometric wake model that can be used to predict production losses of existing and potential wind parks. Motivated by simple engineering wake models, we use wind speed, turbine alignment angle, and distance as explanatory variables. Unlike existing wake models, our approach is mainly data driven and utilizes real world data instead of experimental data. The model is estimated for two wind parks in Germany. We find that the regression model is a good fit with observed wind deficits for both wind parks. The gain in estimation accuracy when switching from a single wake model to a two-wake model is moderate. Compared to the standard Jensen model, a significant reduction of the root mean square error can be attained in the empirical wake model. A scenario analysis revealed that the distance between turbines can be reduced up to three times the rotor size without entailing substantial production losses. In contrast, a suboptimal configuration of turbines with respect to the main wind direction can lead to five times higher production losses.

Our empirical wake model is characterized by several useful features. First, the approach is rather flexible and can be easily estimated for any wind park design. Second, due to its data driven nature, the model is able to capture specific effects that occur in existing wind parks. In contrast to the Jensen model, which only shows two adjustable parameters (thrust coefficient and surface roughness), the regression model has much more parameters that can be tailored to specific wind parks. This advantage, however, comes at the cost of a reduced generalizability of the results: an estimated set of regression parameters cannot be used to assess wake effects in a different environment, e.g. for different turbine types or at a completely new location. The model is rather useful to predict the effects of extending existing wind parks or changing wind conditions. Other limitations of our model come from the fact that it produces meaningful results for only a limited range of turbine alignment angles and that discontinuities occur when mixing single and two-wake models.

There are several starting points for extending and refining the proposed empirical wake model. First of all, it would be relevant to test the model's performance for other wind parks under a variety of external conditions using other data sources, particularly meteorological mast data. From a methodological viewpoint, alternative definitions of relevant turbines, the inclusion of nonlinear terms in the explanatory variables, or the application of a Tobit regression that incorporates non-disturbing turbines into the model could further improve the model results.

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